IFRS 9 ECL Scenario Weighting

A benchmark model

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IFRS 9 ECL: What are we supposed to do?

5.5.17 An entity shall measure expected credit losses of a financial instrument in a way that reflects:

(a) an unbiased and probability-weighted amount that is determined by evaluating a range of possible outcomes;

(b) the time value of money; and

(c) reasonable and supportable information that is available without undue cost or effort at the reporting date about past events, current conditions and forecasts of future economic conditions.

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Basis for conclusion on scenario usage

- BC5.264: In the IASB's view, an **expected value measurement** is the most relevant measurement basis because it provides information about the timing, amounts and uncertainty of an entity's future cash flows. This is because an expected value measurement would:

- "an entity will be required to **consider multiple scenarios and possible outcomes and their probability** of occurrence."

- b) "reflect that the pricing of financial instruments includes the consideration of expected credit losses"

- c) "not revert (at any time) to an incurred credit loss model—**all financial instruments have risk of a default occurring**"

- d) "have the same objective regardless of whether an entity performs the measurement at an **individual or a portfolio level.**"

- E) "provide useful information to users of financial statements (ie information about the **risk that the investment might not perform**)."

Range of possible scenarios

- Why are multiple scenarios needed?

Scenario	Unemployment rate	Scenario probability	Associated ECL
Alternative A	4%	33%	75
Base case	**6%**	**34%**	**100**
Alternative B	8%	33%	275

100-75 = 25

275-100 = 175

175 > 25
Downturn increase losses more

Base case ECL ≠ **Probability-weighted ECL**

- Base case forecast of unemployment rate = 6%
- ECL given employment rate of 6% = CU100

- Probability-weighted ECL = CU150, i.e. (33% x 75)+(34% x 100)+(33% x 275)

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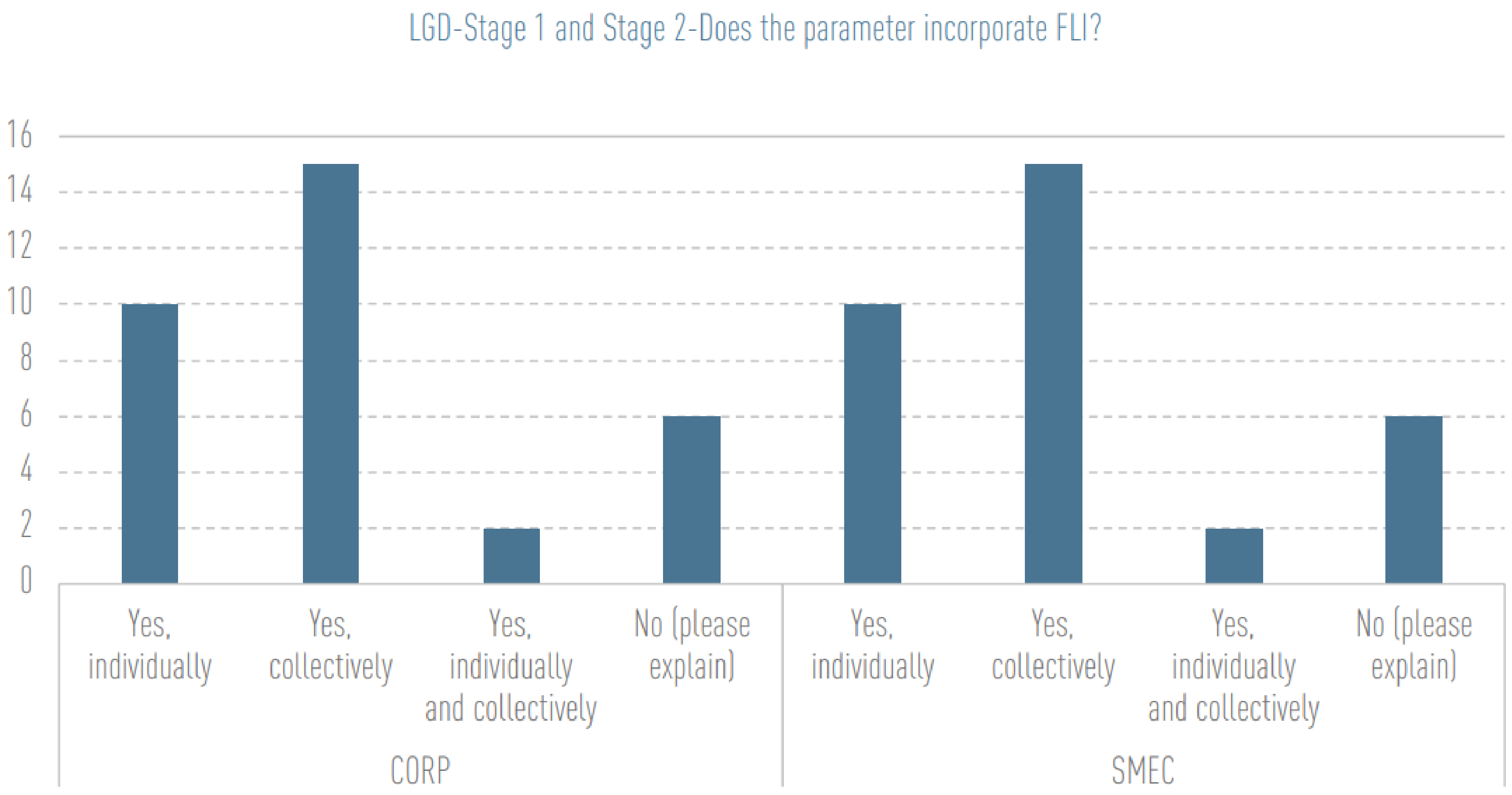
Range of possible scenarios: PD & LGD

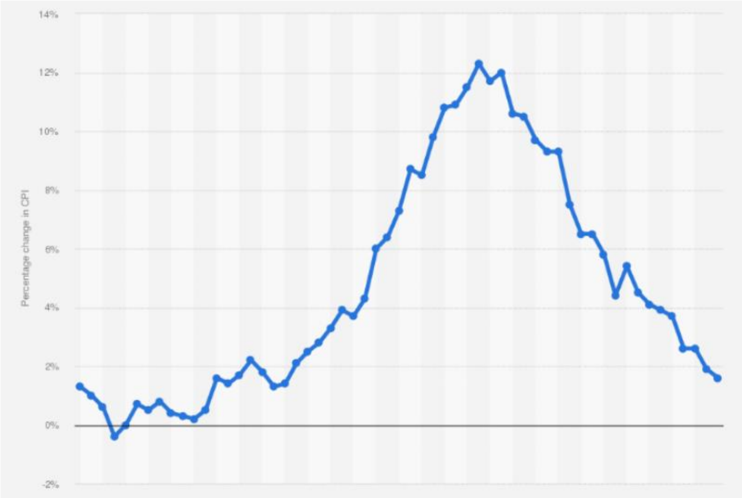


PD & LGD: Why the dynamic matters

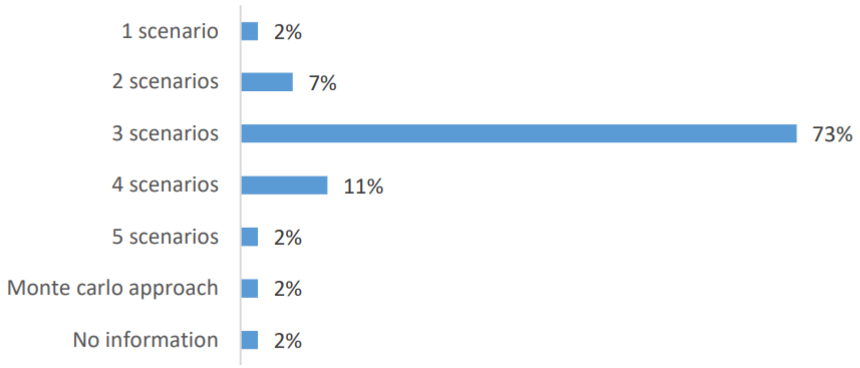
- An issue is that PD and LGD outcomes are often correlated:

$$\mathbb{E}[PD \times LGD] = \mathbb{E}[PD] \times \mathbb{E}[LGD] + \mathbb{C}[PD, LGD]$$

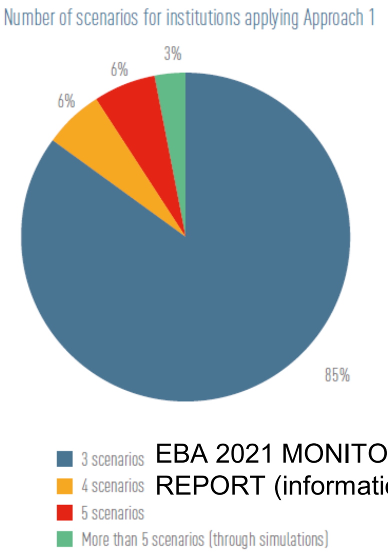
- Example: During the inflation period in Sweden:
  - Households struggled with additional costs: Higher default rate
  - Debt collection companies got increased funding costs: Higher LGD



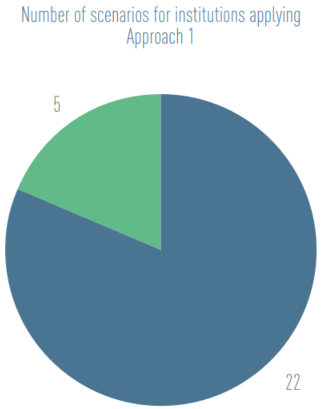
Range of possible scenarios



ESMA 2021: On the application of the IFRS 7 and IFRS 9 requirements regarding banks' expected credit losses (ECL)



EBA 2021 MONITORING REPORT (information from 2019)



EBA 2023 MONITORING REPORT

Single scenario + scalar adjustment

Probability-weighted ELC based on 'n' number of scenarios

Monte Carlo simulation

Requires some form of multi-scenario for benchmark

Most common "Goldilocks" choice

Most theoretically correct, practically complex

5.5.18 When measuring expected credit losses, an entity need not necessarily identify every possible scenario. However, it shall consider the risk or probability that a credit loss occurs by reflecting the possibility that a credit loss occurs and the possibility that no credit loss occurs, even if the possibility of a credit loss occurring is very low.

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5.5.17 An entity shall measure expected credit losses of a financial instrument in a way that reflects:

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“Probability weighted”

- BC5.265: “…When there are many possible outcomes, an entity can use a **representative sample of the complete distribution** for determining the expected value. The main objective is that **at least two outcomes are considered**: the risk of a **default** and the risk of **no default**.”

- The goal is to compute an expectation value:

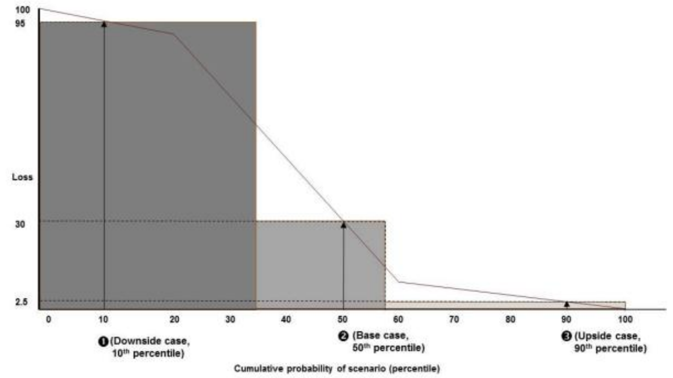
$$ECL = \int ECL(z)p(z)dz = \mathbb{E}[ECL(z)]$$

- “z” represents a simplified single variable macro factor (compare with IRB)

- As discussed above, reliant on n values of z.

$$\int ECL(z)p(z)dz \approx \sum_{k=1}^{n} w_k ECL(z_k)$$

- w_k is **not** the “probability” of scenario k (probability is 0)
- It is a weight that tries to reproduce the integral



Using this approach, the ECL would be estimated as being the sum of:

- 95 (loss in downside scenario) × 34% weighting = 32.3
- 30 (loss in base case scenario) × 23% weighting = 6.9
- 2.5 (loss in upside scenario) × 43% weighting = 1.1

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Monte-Carlo vs finite number scenarios

- If the Monte-Carlo method is selected, n scenarios will be generated

$$\int ECL(z)p(z)dz \approx \sum_{k=1}^{n} w_k ECL(z_k)$$

- Advantage of Monte-Carlo is that the weights directly can be set as $w_k = \frac{1}{n}$
- If one look at the expected distribution of n random scenarios, they will be in percentiles $\frac{i}{n+1}$
- Example, with 3 scenarios this would be 25 %, 50 % and 75 % percentile

- However, it is known for Monte-Carlo theory that one can improve convergence by selecting "rarer" scenarios and adjusting down the weights of the extreme scenarios.
- Principle **of importance sampling**. Idea is that less likely scenarios could have major important.
- Even with few scenarios, the question is if one can make some more sophisticated selection

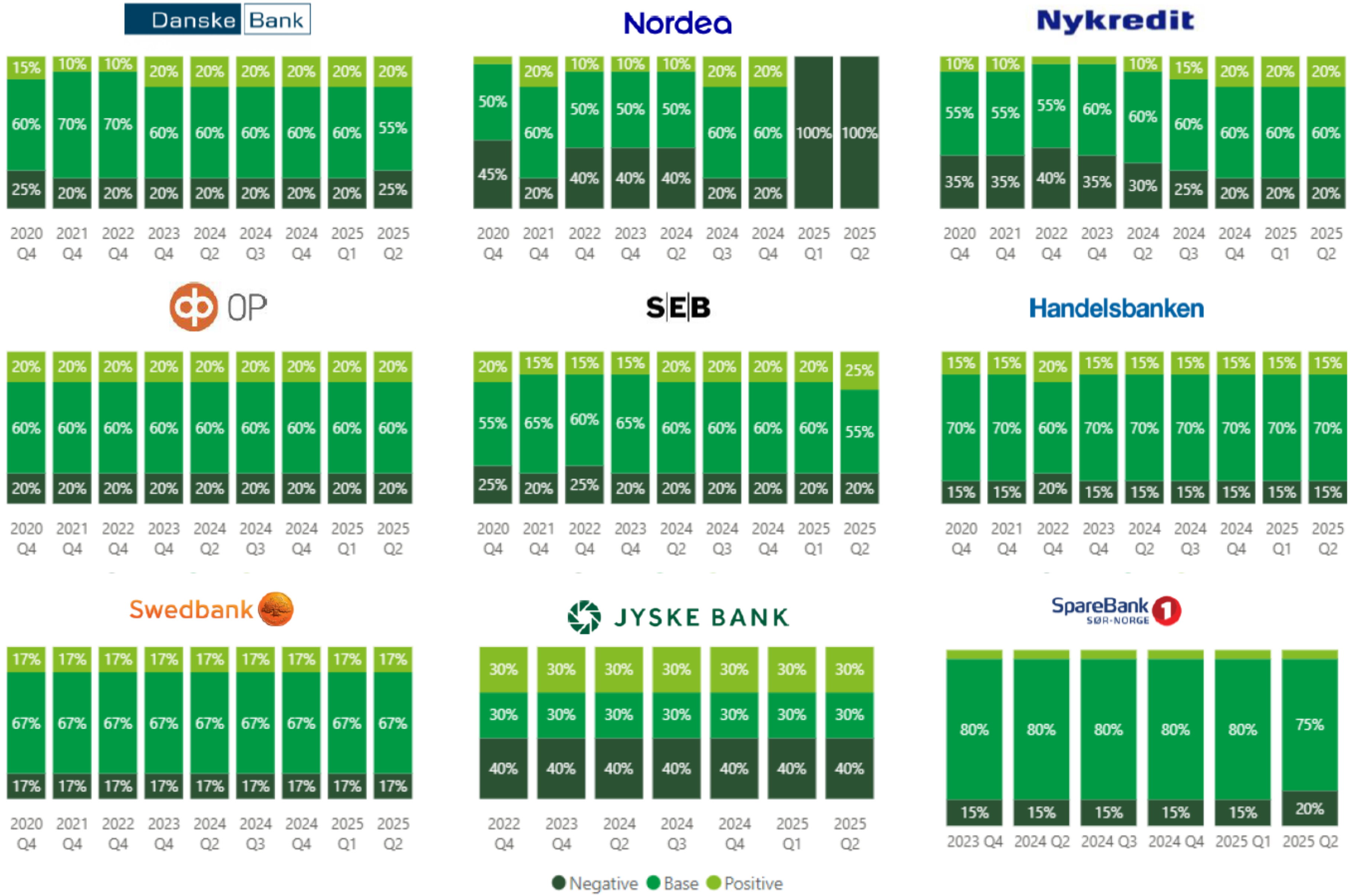
How to select weights: Extreme example

- IFRS 7 disclosures practice is to show weights and the effect of weight = 100 % for a single scenario
- An extreme example is the Swedish niche bank TF bank in 2023:

MACROECONOMIC SCENARIOS
The table below applies to both the Group and the Parent company

Scenario	Probability of occuring	Impact on provi-sions (stage 1)	Probability weitgh-ted outcome
Adverse	1%	50 % increase	
Positive	20%	10 % decrease	98,5%
Base	79%	Neutral	Lower than base case

Benchmarking of Nordics banks by Deloitte



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The convexity effect

- A common question is why $\int ECL(z)p(z)dz$ is not just equal to $ECL(0)$ if the underlying distribution is symmetric.
  - The “upturn and downturn scenario effects cancel”

- One can Taylor series expand: $ECL(z) = ECL(0) + \frac{dECL}{dz}(0)z + \frac{1}{2}\frac{d^2ECL}{dz^2}(0)z^2 + \frac{1}{6}\frac{d^3ECL}{dz^3}(0)z^3 + O(z^4)$
- If substituted into the integral, gives

$$\int ECL(z)p(z)dz = \int \left[ECL(0) + \frac{dECL}{dz}(0)z + \frac{1}{2}\frac{d^2ECL}{dz^2}(0)z^2 + \frac{1}{6}\frac{d^3ECL}{dz^3}(0)z^3 + O(z^4)\right]p(z)dz \approx ECL(0) + \frac{1}{2}\frac{d^2ECL}{dz^2}(0)$$

Single scenario + scalar adjustment

- The ECL scenario dependence is to first order driven by convexity, downturn scenarios are more severe than upturn scenario, even at symmetric percentiles: Jensen’s inequality

- Since convexity is positive, one would expect the weighted ECL to be higher than base case.

The Taylor solution

- Assume you only have access to three scenarios that are Δ close to each other. $z = -\Delta \quad z = 0 \quad z = \Delta$
- We can then use the second order approximation to obtain a weight solution:

$$\int ECL(z)p(z)dz \approx ECL(0) + \frac{1}{2}\frac{d^2ECL}{dz^2}(0) \approx ECL(0) + \frac{1}{2}\left[\frac{ECL(\Delta) - 2ECL(0) + ECL(-\Delta)}{\Delta^2}\right]$$

$$= ECL(0)\left(1 - \frac{1}{\Delta^2}\right) + ECL(\Delta)\frac{1}{2\Delta^2} + ECL(-\Delta)\frac{1}{2\Delta^2}$$

- Weights are $1 - \frac{1}{\Delta^2}$ for base case and $\frac{1}{2\Delta^2}$ for the "upturn" and "downturn" solution
- They sum to 1.
- In the limit $\Delta \to 0$ we get back the second order expansion.

- For $\Delta < 1$, $1 - \frac{1}{\Delta^2} < 0$ so base scenario weight will be negative
- Good example on the difference between weights and probabilities.

Weights needed for both ECL and SICR

- If the integral $\int ECL(z)p(z)dz$ should be optimized, to what degree should the choice depend on the function $ECL(z)$?

- In the same vein, should the scenario weights change with the macro economic environment?

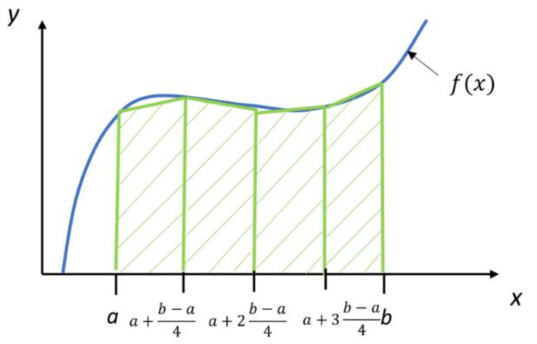
- One problem is that the weights are needed in two different contexts:
  - Calculation of $\int ECL(z)p(z)dz$ for the ECL calculation
  - Calculation of $\int PD(z)p(z)dz$ for the SICR calculation (since stage choice is same in all scenarios)

One financial instrument cannot exist in stage 1 and in stage 2 at the same time.

- Hence it can be preferable to find a generic weight choice that work independent of function

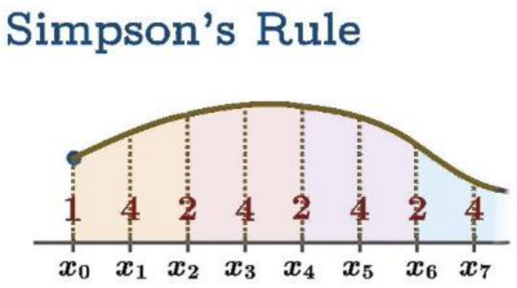
Approximation strategy: Zero and odd orders

- In deriving general quadrature rules such as trapezoidal integration and Simpson's rule a starting point is to look at a monomial basis $1, z, z^2, z^3, \ldots$



- If one apply the same recipe for 3 scenarios, one obtain
- Zero order:

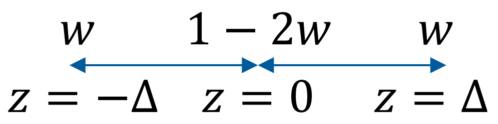
$$1 = \int p(z)dz = 1w_{base} + 1w_{upturn} + 1w_{downturn}$$



- Interpretation: Weights should add up to 1 to be normalized.
- Odd orders:

$$0 = \int z^{2n+1}p(z)dz = z_{upturn}^{2n+1}w_{upturn} + z_{downturn}^{2n+1}w_{downturn}$$

- Symmetric percentile choice is made, $z_{upturn} = \Delta, z_{downturn} = -\Delta$, imply weights for upturn and downturn scenarios should be the same.
- $w_{base} = 1 - 2w$



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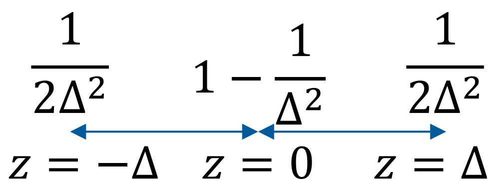
Approximation strategy: Second order

- For the second order, $\int z^2 p(z)dz = 1$ simply by recalling z, hence it can be seen as a way to define the scale of the z variable.

Relationship between stress severity and weight

- Carrying out the evaluation to second order gives:

$$1 = \int z^2 p(z)dz = \Delta^2 w + (-\Delta)^2 w = 2w\Delta^2$$



- Solving gives $w = \frac{1}{2\Delta^2}$, and $w_{base} = 1 - \frac{1}{\Delta^2}$
- This is the Taylor solution discussed above but with no restriction on size of Δ
- If $\Delta > 1$ (reasonably diverse scenarios selected), weights will all be positive.

- To translate Δ into percentile, need to set the probability distribution p.
- A choice consistent with the IRB Vasicek model is to use a normal distribution.
- Percentiles for outer scenarios become $\Phi(\pm\Delta)$

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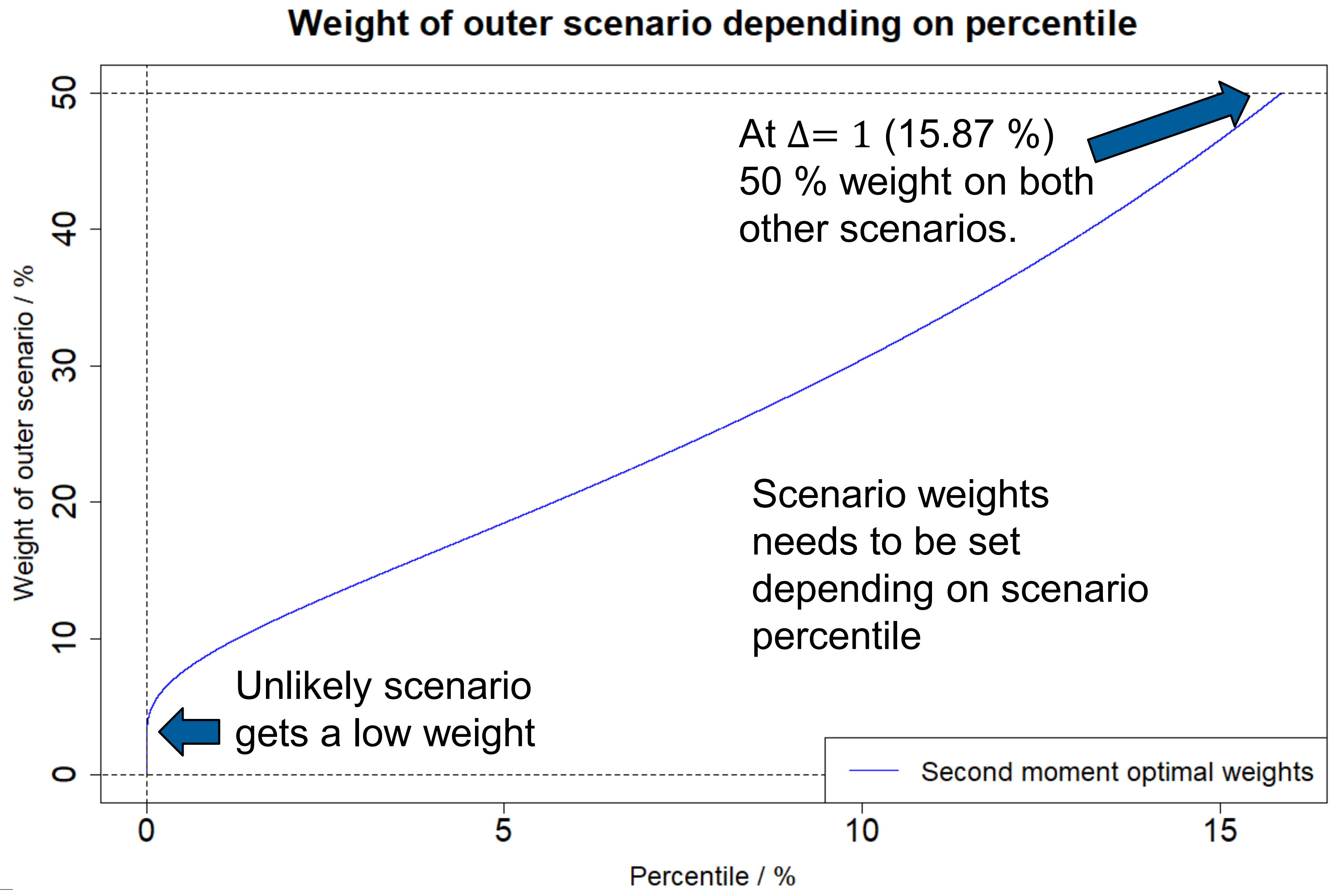
Relationship between percentile and weight



Benchmark model from IRB

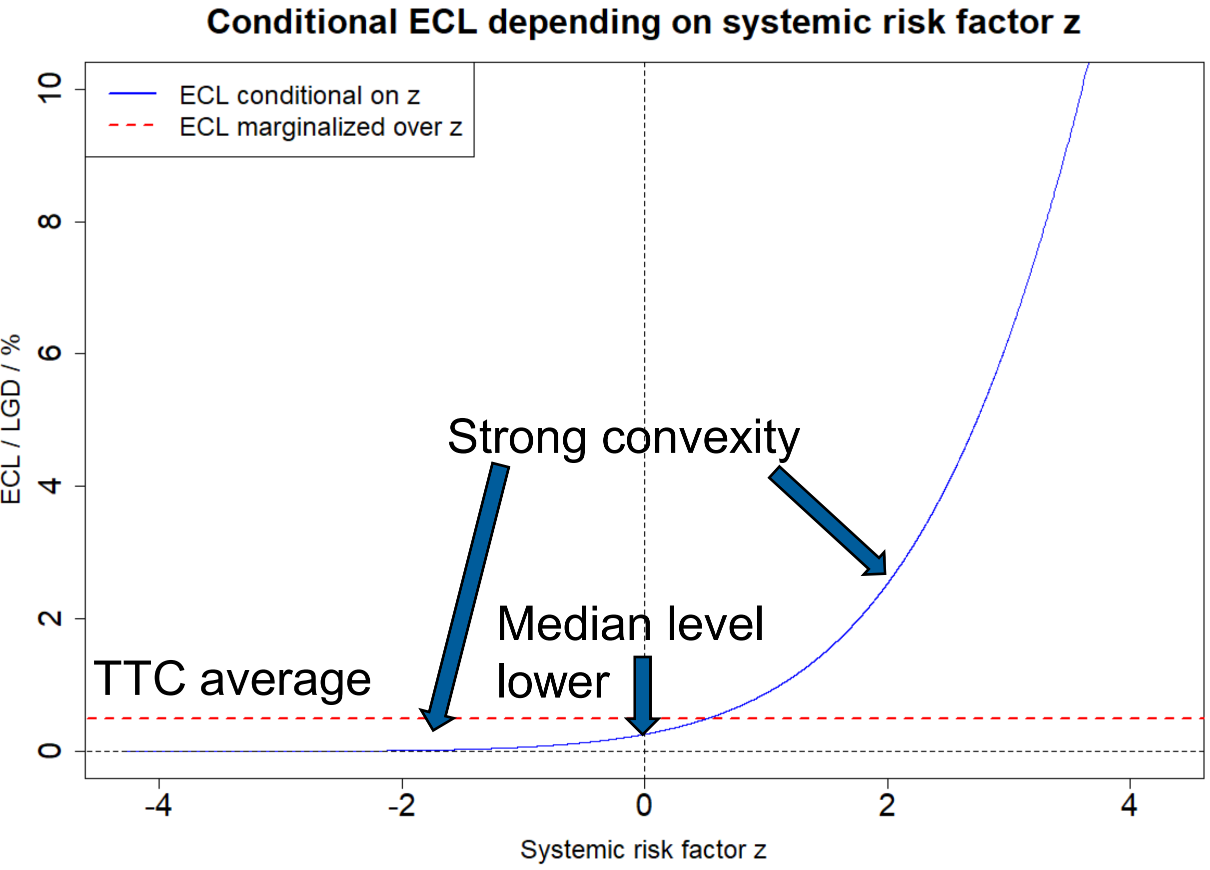
- We want to be able to put the percentile and weights to the test.

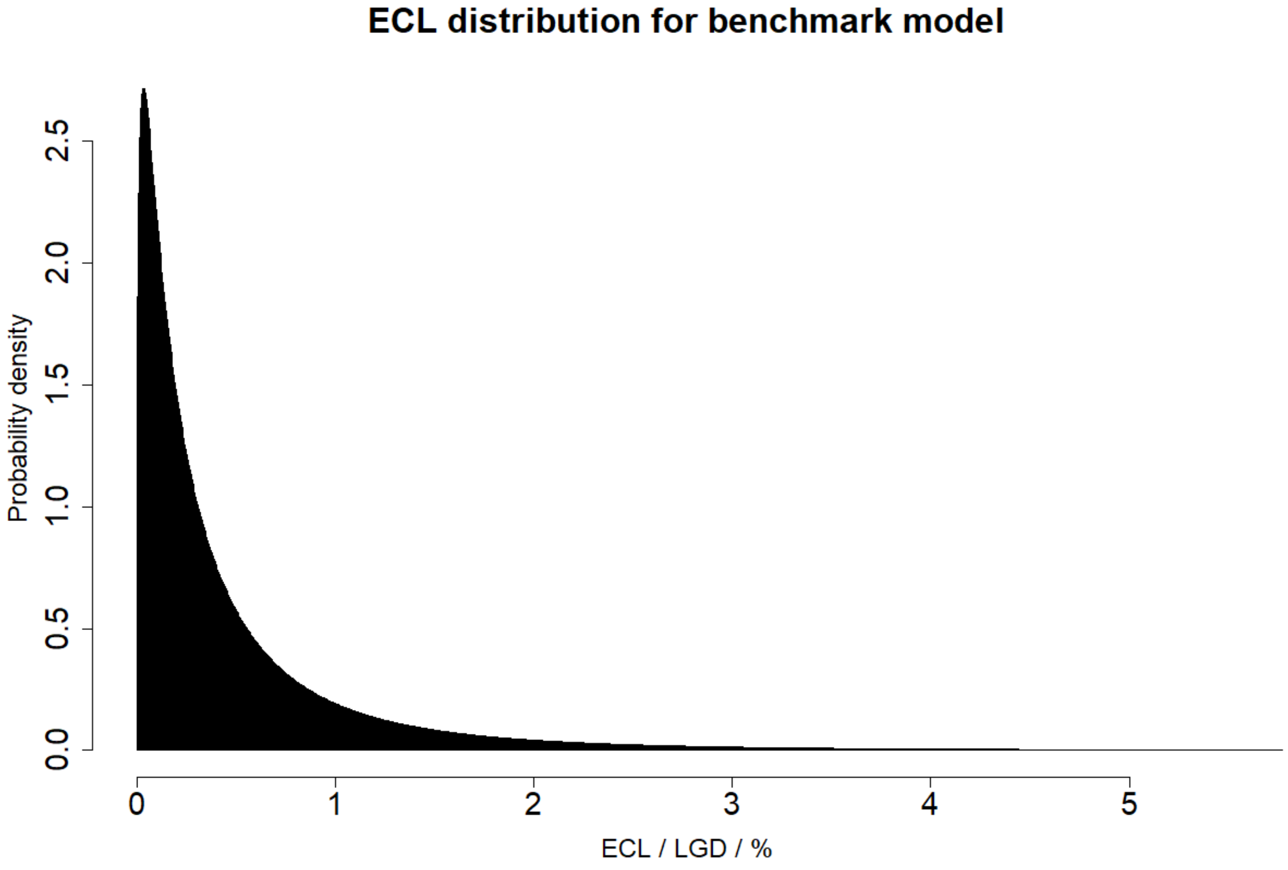
- A simple benchmark model can be constructed from IRB.
- Look at a mortgage portfolio with a TTC PD of $PD_{TTC} = 50 \text{ bp}$
- Constant LGD is used both for simplicity and the lack of a simple LGD macro benchmark model
- This means that the long-term average PD is 50 bp. However, over the economic cycle it will both be higher and lower.
- The asset correlation R (15 % for mortgages) describe the responsiveness of the PiT PD to the systemic risk factor z.

- The Vasicek model tells us:

$$PD(z) = \Phi\left(\frac{\Phi^{-1}(PD_{TTC}) + \sqrt{R}z}{\sqrt{1-R}}\right)$$

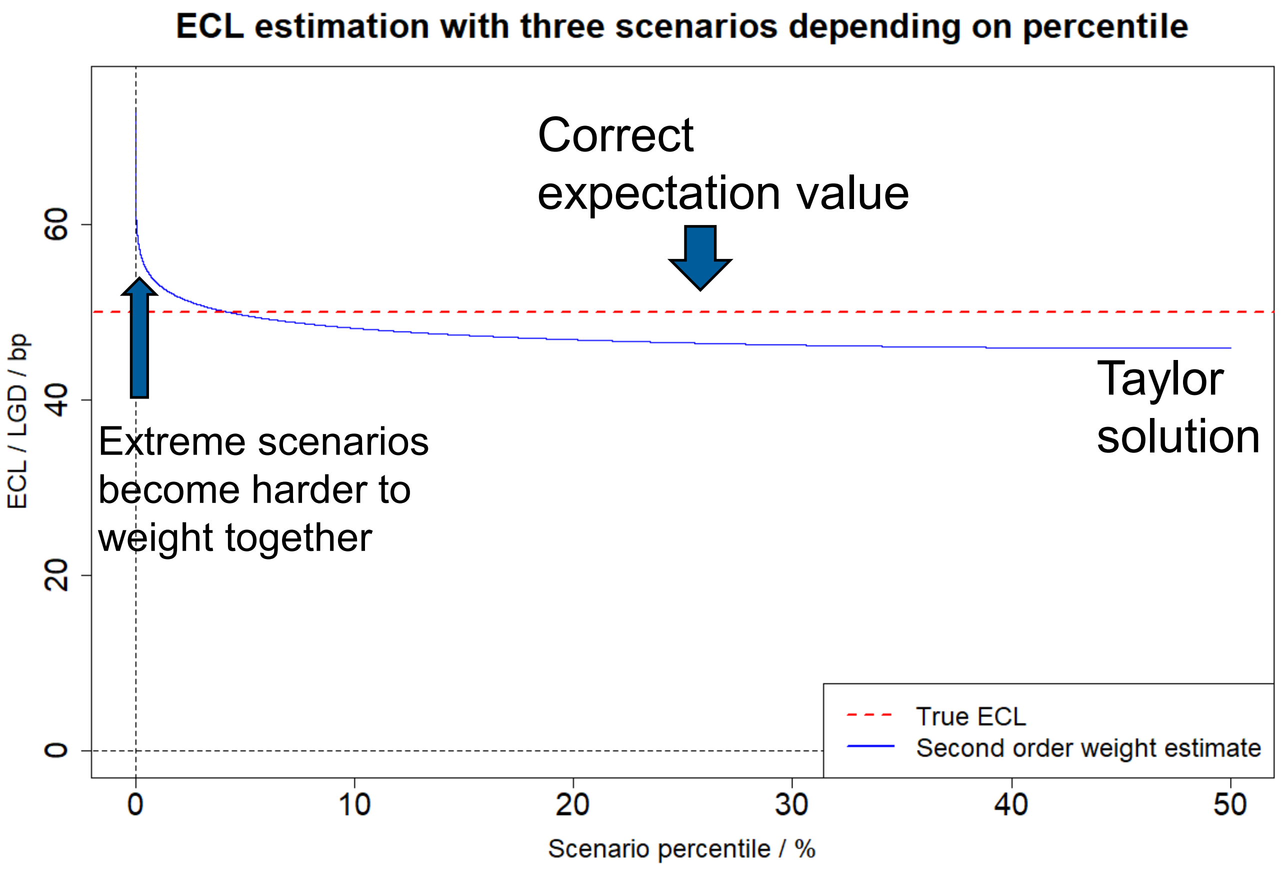
ECL distribution in the benchmark model





Applying the scenario weighting to the benchmark model

- In the benchmark model, $\mathrm{ECL} = \mathrm{LGD} \int PD(z)\, p(z) dz$
- We assume here that currently we are at a point in the economy where z = 0 is best estimate
- $p(z)$ will then be standardized normal
- A practical feature of the Vasicek model is that $\int PD(z)p(z)\, dz = PD_{TTC}$, so closed form solution
- Computing $\int ECL(z)p(z)\, dz$ using three scenarios gives:



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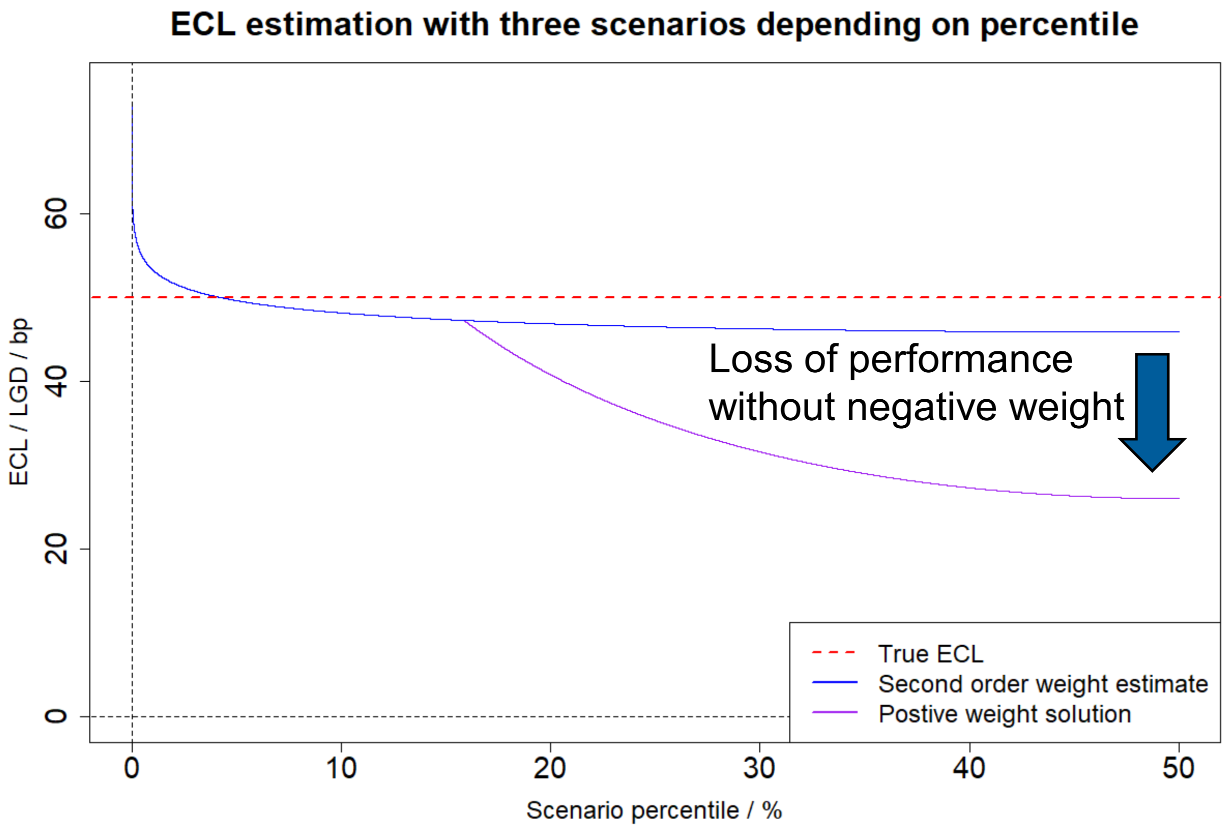
Negative weights: Improving accuracy

- In deriving the solution, we noted that it is possible for weights to become negative
- In the benchmarking, we have introduced the restrictions that all weights must be positive
- Beyond where negative weights would appear, the positive weight solution has considerable worse accuracy



- For $\Delta \rightarrow 0$, the negative weight solution converges to $ECL(0) + \frac{1}{2}\frac{d^2ECL}{dz^2}(0)$
- The positive solution will be $\frac{ECL(-\Delta)+ECL(\Delta)}{2}$
- It will converge to $ECL(0)$ in the limit $\Delta \rightarrow 0$, missing the convexity contribution
- Standard speaks about “probability-weighted” amount, complex to introduced negative

Avoid the issue by selecting diverse enough scenarios

Accuracy of ECL estimate depending on Δ

- From the benchmark model we can draw a number of conclusions.

- Changing the weighting with the percentile allow us to overall with reasonable accuracy estimate the expectation value.

- Picking very extreme scenarios are hard to combine together, extreme tails are not representative.
  - With $\Delta \gg 1$, estimates becomes very extreme scenario $(PD \approx 100\ \%)$ but also very unstable since if $\Delta \to \infty$, PD can at most be 100 %, but $w = \frac{1}{2\Delta^2} \to 0$, so weighted ECL will be just baseline ECL again (effect of scenario usages disappear)
- Letting $\Delta \to 0$ converge to finite accuracy, but inefficient as all three scenarios will be very similar, so limited opportunity to investigate the full behaviour of the distribution.

- Numerically, the percentile 4.26 % is the optimal choice for the specific model by simply computing which scenarios that give exactly the right ECL value.

Right-sizing the percentile

- Is it possible to compute an estimate for the optimal percentile choice?
- A simple solution is to move on to fourth order, kurtosis:

$$\int z^4 p(z)dz = 3$$ Kurtosis for a normal distributed variable

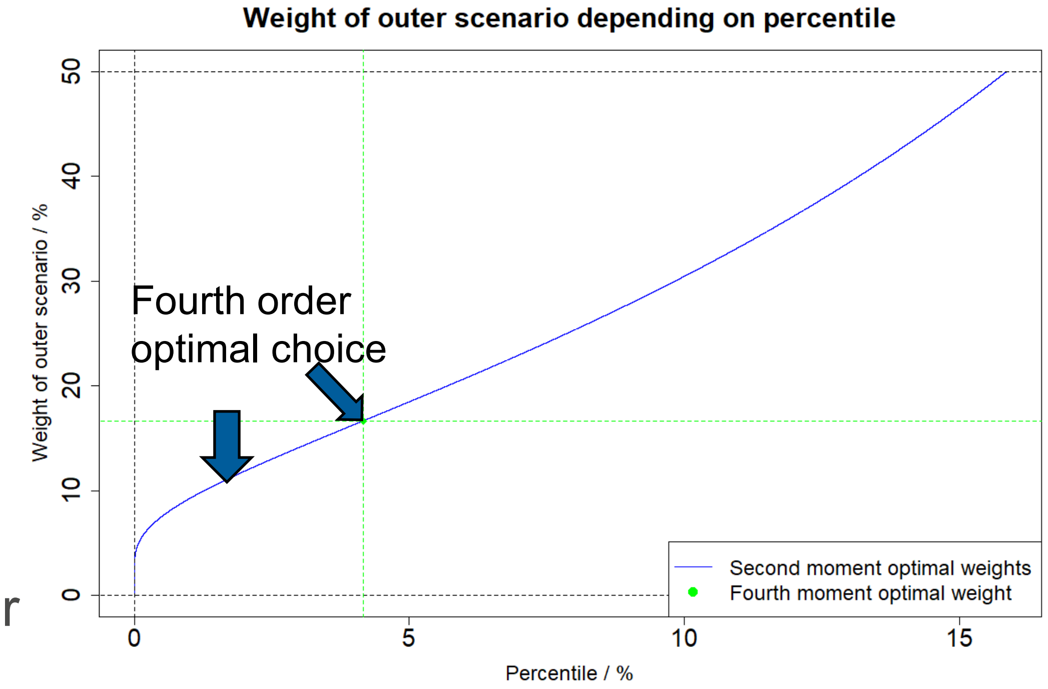
- This is a property of the normal distribution (that is used for the probability density p)
- Setting up the estimate

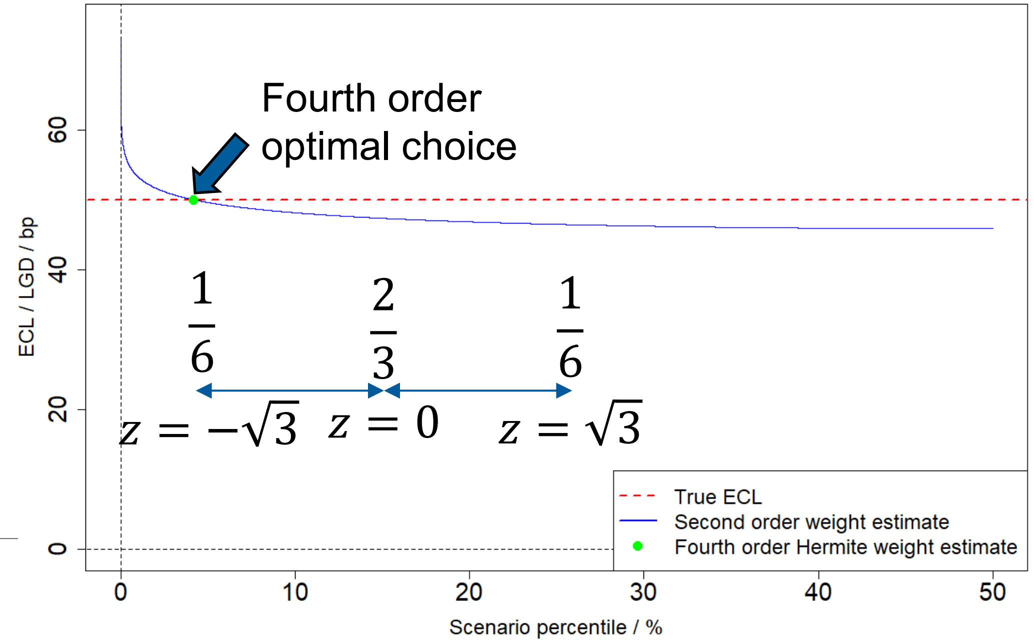
Using the second order relationship for the weight

$$3 = \int z^4 p(z)dz = \Delta^4 w + (-\Delta)^4 w = 2w\Delta^4 = \Delta^2$$

$$1 = \int z^2 p(z)dz = w\Delta^2 + w(-\Delta)^2 = 2w\Delta^2$$

- Optimal Δ based on fourth order is $\sqrt{3}$, corresponding to **percentile 4.16 %** and **weight 1/6**, close to the numerical answer for the benchmark model and with a relative error of 0.106 %



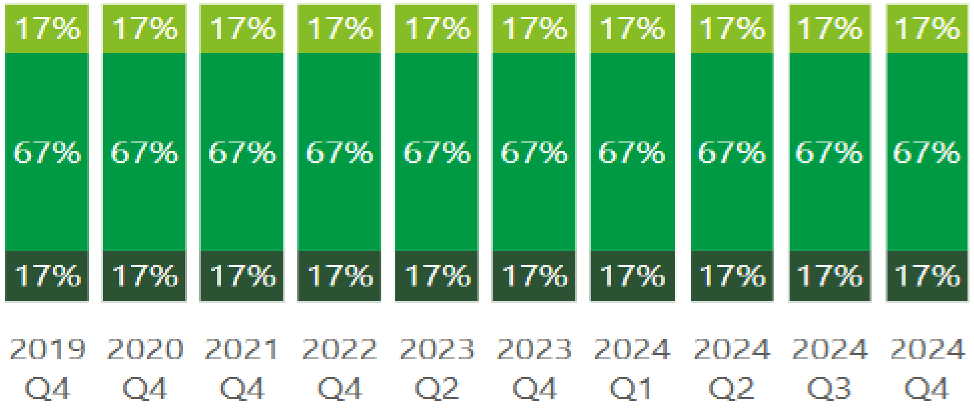


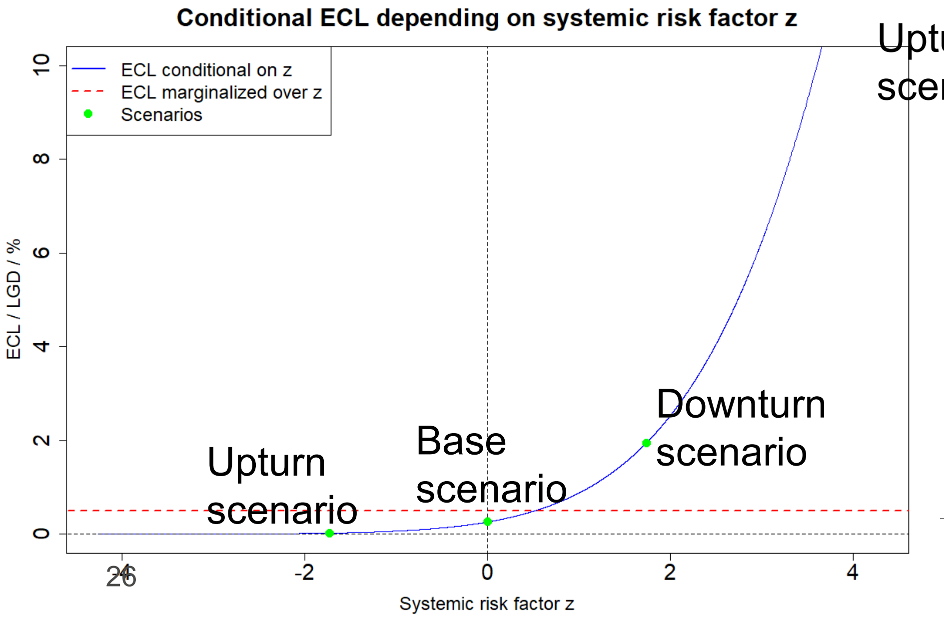
Summary of the three-scenario case

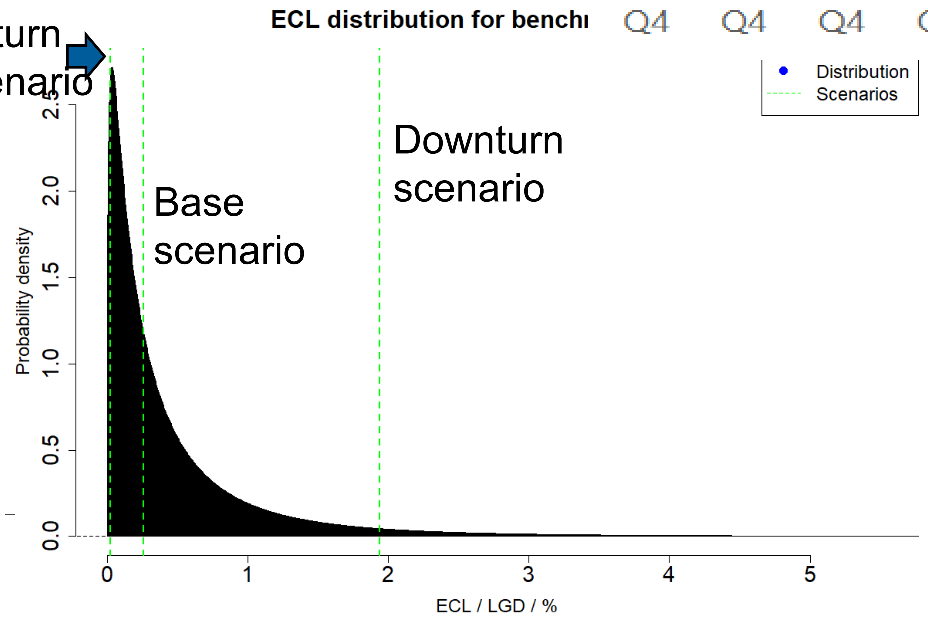
- Assume that the probability distribution of the systemic variable is normal, $p(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
- Symmetrical outer scenarios at percentile 4.2 %. Weights

Scenario	Percentile	Weight
Upturn	$\Phi(-\sqrt{3}) \approx 4.2\ \%$	$1/6 \approx 17\ \%$
Base	$\Phi(0) = 50\ \%$	$2/3 \approx 67\ \%$
Downturn	$\Phi(\sqrt{3}) \approx 95.8\ \%$	$1/6 \approx 17\ \%$

Swedbank







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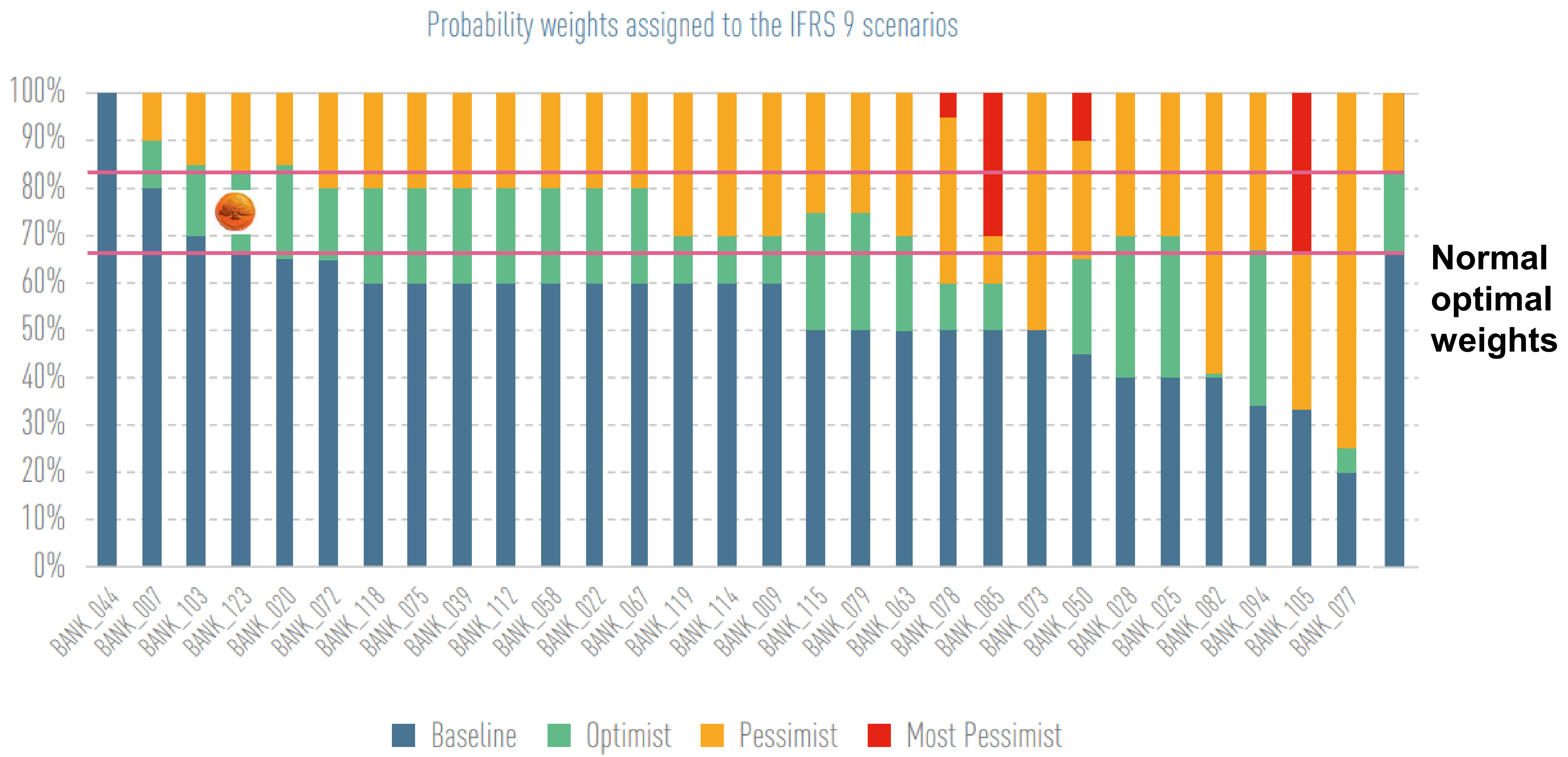
Benchmarking the normal optimal weights



Should the weights change during economic cycle?

- Notice that the weights we have derived has been independent of the specific ECL function, as well as only requiring using to having a normal uncertainty distribution around the forecast.

- In the benchmark model this requirement for normality is preserved no matter where in the economic cycle we are
  - In the analysis we have assumed the distribution is centred around $z = 0$, but analysis should have been repeated at any point due to the gaussian copula.

- The general advice of keeping weights constant is argued by for example Macro Folpmers at Deloitte

When to Change IFRS 9 Scenario Weights for ECL: A Simple Rule

- If for example downturn weights are increased as the risk in the scenarios increases, this will lead to an over-correction (double counting the downturn effect) and pro-cyclicality

- Let the weights stay constant and let the scenario outcomes change with the economy!

Multiple risk factor scenarios

- So far, we have had 1 risk factor. With the discussion on emerging risk, one can imagine looking at multiple risk factors (Economic, climate, geopolitical).

- Denote the risk factor vector $\boldsymbol{z} = (z_1, z_2, \dots)$ of m different risks
  - If factors are correlated, can be turned uncorrelated by spectral transformation.
  - ECL is a function of the joint risk factor $ECL(\boldsymbol{z})$ (Interactions also possible)
  - Normal independence-transformed factors have density $p(\boldsymbol{z}) = \prod \left[\frac{1}{\sqrt{2\pi}} e^{-z_i^2/2} \right]$

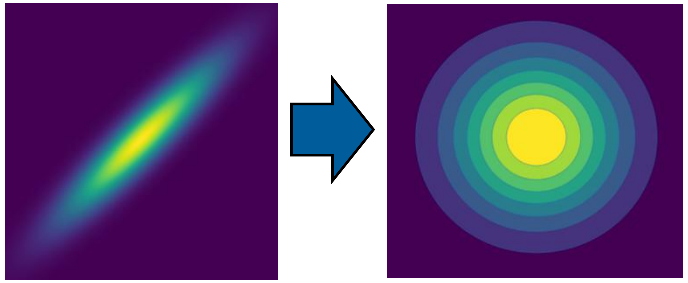
- Density per risk factor replaced by

$$\frac{1}{\sqrt{2\pi}} e^{-z_i^2/2} \rightarrow \frac{2}{3}\delta(z_i) + \frac{1}{6}\left[\delta\left(z_i - \sqrt{3}\right) + \delta\left(z_i + \sqrt{3}\right)\right]$$

- Result is 3^m different scenarios, evaluating all possible combination of the risk factors

- Not really computationally possible for many risk factors, but for two factors we get 9 scenarios

Spectral transformation



$(-\sqrt{3}, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(-\sqrt{3}, \sqrt{3})$
$(0, -\sqrt{3})$	$(0,0)$	$(0, \sqrt{3})$
$(\sqrt{3}, -\sqrt{3})$	$(\sqrt{3}, 0)$	$(\sqrt{3}, \sqrt{3})$

Gauss-Hermite theory: Setup

- There is an alternative way of deriving the optimal percentile and weights using Gauss-Hermite theory.

- The Hermite polynomials $H_0, H_1, H_2, H_3, \ldots$ are defined as orthogonal polynomials such that

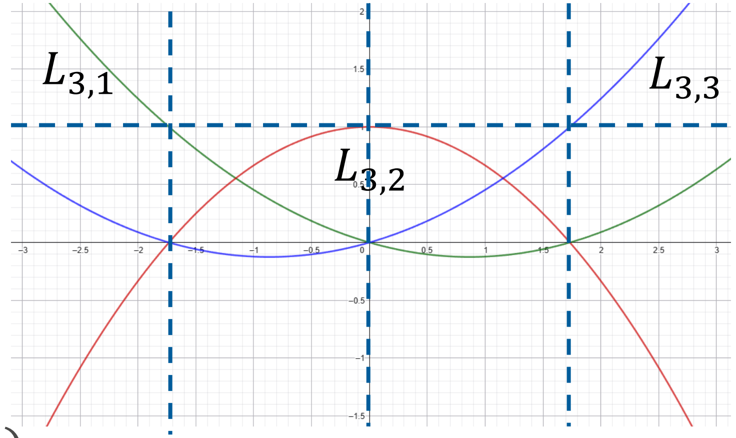
$$\int H_i(z)H_j(z)p(z)dz = \delta_{ij} = \begin{cases} 1, \text{if } i = j \\ 0, \text{else} \end{cases}$$

Kronecker delta

- Each Hermite polynomial of order n have a set of n Lagrange polynomials $L_{n,i}$ on the form

$z_1, z_2, \ldots, z_n$ are roots of H_n

$$L_{n,i}(z) = \prod_{j=1, j\neq i}^{n} \left(\frac{z - z_j}{z_i - z_j}\right)$$



- The idea behind this construction is that $L_{n,i}(z_j) = \delta_{ij}$.
- Then any polynomial f of order of order 2n-1 can be written:

$$f(z) = \sum_{i=1}^{n} f(z_i)L_{n,i}(z) + \sum_{i=0}^{n-1} c_i H_i(z)H_n(z)$$

Lower order terms   Higher order terms

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Approximating ECL(z) with a polynomial

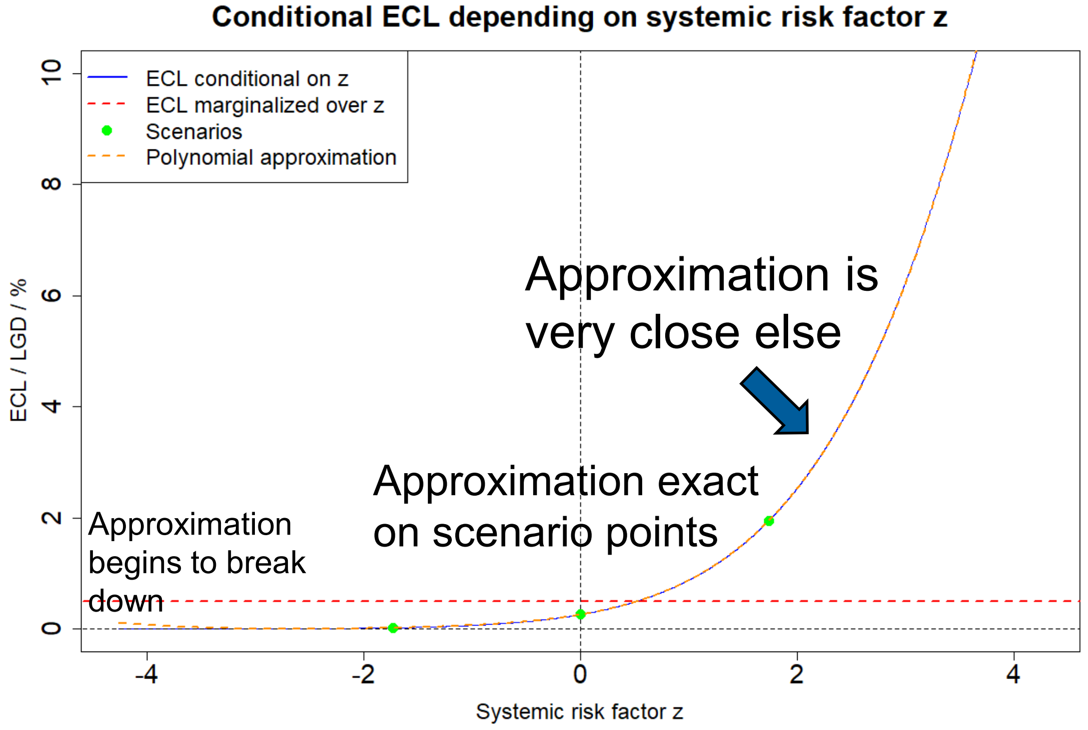
- The idea in how to select the weights is that a polynomial can be used to approximate the $ECL(z)$ function.
- The approximation $\widehat{ECL}(z)$ will be a fifth order polynomial, how well could such replicate the benchmark ECL model?

$$\widehat{ECL}(z) = \sum_{i=1}^{n} ECL(z_i)L_{n,i}(z) + \sum_{i=0}^{n-1} c_i H_i(z) H_n(z)$$

- Coefficients for lower order term directly from ECL
- Coefficients from higher order terms selected to minimize L^2 norm:

$$\int \left[ECL(z) - \widehat{ECL}(z)\right]^2 p(z)dz$$

- Numerical integration of the integrals gives system of equations $Ac = b$



$$A_{ij} = \int H_i(z) H_j(z) H_n^2(z) p(z)dz$$

$$b_i = \int \left[ECL(z) - \widehat{ECL}(z)\right] H_i(z) H_n(z) p(z)dz$$

Gauss-Hermite theory: The Magic

- We want to compute the integrals of the form $\int f(z)p(z)dz$. Substituting the Hermite representation gives:

$$\int ECL(z)p(z)dz \approx \int \widehat{ECL}(z)p(z)dz = \int \left\{ \sum_{i=1}^{n} ECL(z_i)L_{n,i}(z) + \sum_{i=0}^{n-1} c_i H_i(z)H_n(z) \right\} p(z)dz$$

$$= \sum_{i=1}^{n} \left[\int L_{n,i}(z)p(z)dz \right] ECL(z_i) + \sum_{i=0}^{n-1} c_i \left[\int H_i(z)H_n(z)p(z)dz \right] = \sum_{i=1}^{n} w_i ECL(z_i)$$

0, since $i \neq n$

Gauss-Hermite quadrature

- Here one has introduced the w_i as the integrals $w_i = \int L_{n,i}(z)p(z)dz$
  - Weights are independent of the ECL function.
- We see that for polynomials up to 2n-1, the higher order terms have no impact, only the function values at the roots and the associated weights mattered (c_i anyway hard to compute)
- Hence in practice there is a simple recipe to approximate the integrals if we want **n** scenarios
  - 1. Find the roots of the n:th Hermite polynomial
  - 2. Compute the weight integrals $w_i = \int \prod_{j=1, j \neq i}^{n} \left(\frac{z - z_j}{z_i - z_j} \right) p(z)dz$

Gauss-Hermite theory: corollaries

- It is interesting to note that a number of the properties we have derived come directly from the Gauss-Hermite theory:

- The scenario positions z_i are the roots of H_n.
  - Strum-Liouville theory show that all roots have multiplicity 1. This means that an even Hermite polynomial can never include the base case z = 0
  - It also shows that the Hermite polynomials must be either **even** or **odd**.
  - Hence **if base scenario** should be included, an **odd number** of scenarios must be selected
  - Question the practice of using both a pessimist and most pessimist scenario without symmetry

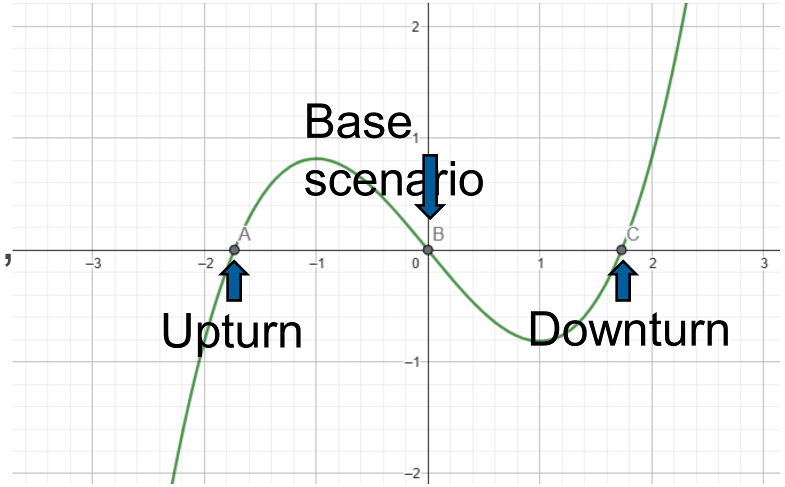
Baseline Optimist Pessimist Most Pessimist

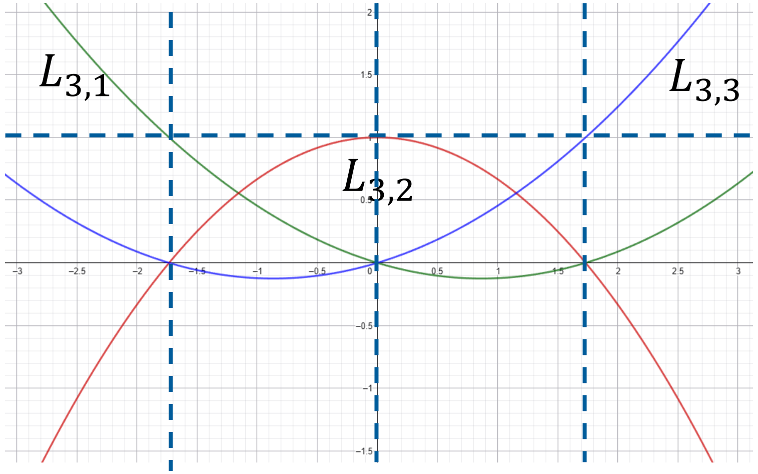
- The odd Hermite polynomials will always have **symmetrical roots** around the origin
  - Argument for selecting the scenarios and weights symmetrically

- The weights will **always add to 1**, since $\sum_{i=1}^{n} w_i - 1 = \sum_{i=1}^{n} \prod_{j=1, j\neq i}^{n} \left(\frac{z-z_j}{z_i-z_j}\right) - 1$ is a n-1 polynomial with n roots, so it is always zero by fundamental theorem of algebra.

Practical example for three scenario case

- $H_0(z) = 1,\ H_1(z) = z,\ H_2(z) = \frac{1}{\sqrt{2}}(z^2 - 1),\ H_3(z) = \frac{1}{\sqrt{6}}(z^3 - 3z)$
- If we want 3 scenarios, we need the roots of $H_3(z) = \frac{1}{\sqrt{6}}(z^3 - 3z)$, which is $z = 0, \pm\sqrt{3}$
- The $L_{3,i}(z)$ polynomials are $\frac{1}{6}z(z - \sqrt{3}), 1 - z^2/3, \frac{1}{6}z(z + \sqrt{3})$



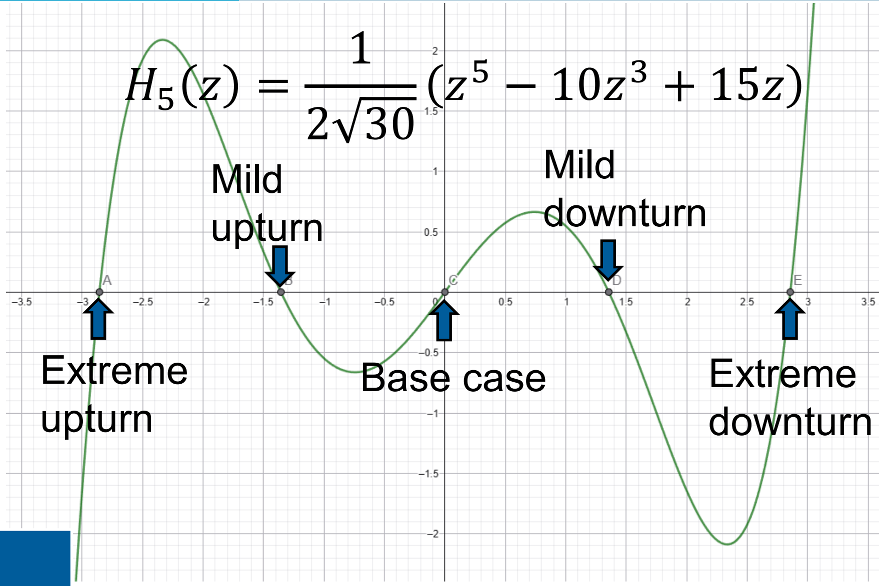


- The weights are given by:
  - $w_1 = \int \frac{1}{6}z(z - \sqrt{3})p(z)dz = \frac{1}{6}\int z^2 p(z)dz = 1/6$
  - $w_3 = w_1 = 1/6$ (by symmetry)
  - $w_2 = 1 - 2w_1 = 2/3$ (by sum of 1 property)

- Reproduce all the results we previously obtained for the three-scenario case

Handelsbanken

Five-scenario case: The next step



- The great value of the Gauss-Hermite theory is that it easily enable us to look at the 5-scenario case.
- It can be shown that the roots are $z = 0, z = \pm\sqrt{5 - \sqrt{10}}, z = \pm\sqrt{5 + \sqrt{10}}$

Scenario	Percentile	Weight
Extreme upturn	$\Phi\left(-\sqrt{5+\sqrt{10}}\right) \approx 0.21\ \%$	$\frac{7}{60} - \frac{1}{3\sqrt{10}} \approx 1.1\ \%$
Mild upturn	$\Phi\left(-\sqrt{5-\sqrt{10}}\right) \approx 8.8\ \%$	$\frac{7}{60} + \frac{1}{3\sqrt{10}} \approx 22\ \%$
Base	$\Phi(0) = 50\ \%$	$\frac{8}{15} \approx 53\ \%$
Mild downturn	$\Phi\left(\sqrt{5-\sqrt{10}}\right) \approx 91.2\ \%$	$\frac{7}{60} + \frac{1}{3\sqrt{10}} \approx 22\ \%$
Extreme downturn	$\Phi\left(\sqrt{5-\sqrt{10}}\right) \approx 99.79\ \%$	$\frac{7}{60} - \frac{1}{3\sqrt{10}} \approx 1.1\ \%$

That scenarios end up between previous roots is again a Strum-Liouville property!

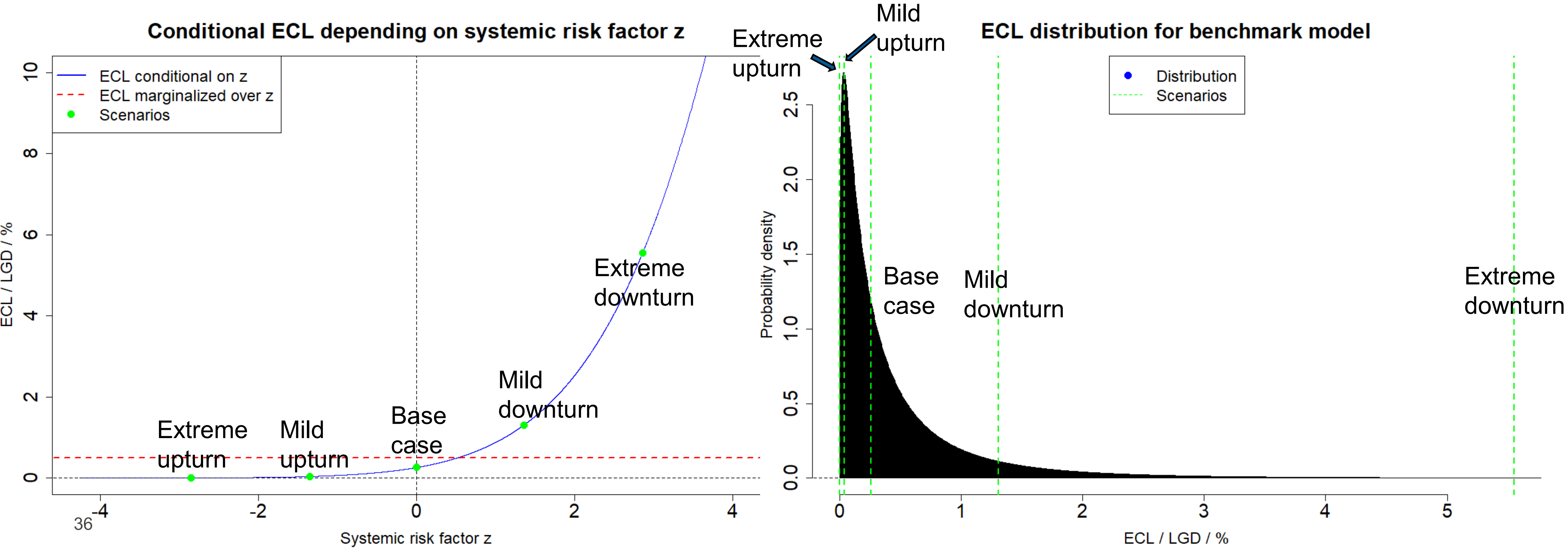
Less weight on base case

Mild downturn less extreme compared to n = 3

Severe downturn

Five-scenario case for the benchmark model

- Going from three scenarios to five scenarios on the benchmark model decrease the relative error from 0.106 % to 0.0016 %, a relative decrease by 98.5 %.
- Five scenario solution is exact up to 9:th order (2 x 5 – 1)
- The extreme scenario connect IFRS 9 with stress testing (1 in 468-year event)



Summary

The non-linearity of ECL with macro economic environment means that the function will be highly convex

→ Scenarios are required to compute the convexity correction to the expectation value.

→ If probability distribution are assumed normal, an optimal three scenario setup can be computed.

→ Gauss-Hermite theory allow to generalize the three scenarios to higher order scenario.

→ Five scenario solution will include both a very severe downturn and upturn scenario linked to stress testing

→ Final reflections: ***While this is not the only way to do IFRS 9 weighting, it serves as a reasonable benchmark for an institution to compare their scenario weighting methodology against.***